Problem 1.

(A)

4-13. A fair coin is independently tossed three times or until a head appears, whichever comes first. Let the random variable X denote the number of heads and let the random variable Y denote the number of tails obtained.

a. Find the probability mass function of X.

b. Find the probability mass function of Y.

c. What is the expected number of heads?

d. What is the expected number of tails?

e. What is the probability that at least two tails are observed?

f. What is the probability that exactly one head is observed?

a. $p_X(X) = 1/6$; y =

(B)

4-18. Consider the random variable Z describing your winnings if you win \$$p$ for each head and lose \$$(1-p)$ for each tail in 3 tosses of a coin with probability p of a head.

a. Find the distribution of Z. Using this distribution, calculate the mean of Z.

b. Write Z as a function of Y, the number of heads. Use your knowledge of the mean and variance of a binomial distribution and the rules for computing means and variances to compute the mean and variance of Z from those of Y. Notice how much easier it is to compute the mean of Z this way.

(C)

4-19. One of the numbers 1-4 is randomly chosen. You are to try to guess the number chosen by asking questions with "yes-no" answers. Suppose your i^{th} question is "Is it i?", $i = 1, 2, 3,$ or 4. Let the random variable Y denote the number of questions required to guess the chosen number. Find

a. The probability mass function of Y.

b. The cumulative distribution function of Y.

c. The mean and variance of Y.

Problem 2.

(A)

4-31. A probability density function for a continuous distribution model has the following form:

$$\begin{aligned} p_Y(y) &= c,\ 0 < y \le 1, \\ &= 0.3,\ 1 < y \le 2, \\ &= 0.1,\ 2 < y \le 4, \\ &= 0,\ \text{otherwise.} \end{aligned}$$

a. What is the value of c?

b. What proportion of the population modeled by this probability density function takes values above 1.5?

(B)

4-32. A population is modeled by the density

$$\begin{aligned} p_Y(y) &= 0,\ y \le 1, \\ &= 1/y^2,\ y > 1. \end{aligned}$$

Based on this model, how much more likely is it that a randomly selected value from the population lies in the immediate vicinity of 2 than in the immediate vicinity of 3?

(C)

4-34. Experience has shown that the width, in mm, of the flange on a plastic connector has the following distribution:

$$\begin{aligned} p_Y(y) &= 50y,\ 0.48 < y < 0.52, \\ &= 0,\ \text{otherwise.} \end{aligned}$$

a. Of the next 1000 connectors produced, how many do you estimate will have widths between 0.50 and 0.51 mm? Show how you arrived at your estimates.

b. How many times as likely is it to produce connectors with flange width close to 0.51 mm as it is to produce connectors with flange width close to 0.49 mm? Justify your answer.

Problem 3.

(A)

4-39. Suppose a set of data approximately follows a normal distribution, and that the mean of the data is 5.1 and the standard deviation is 2.2. If we are willing to model the population from which the data were drawn using the normal distribution, which normal probability density function would you use? What would be your estimate of the proportion of population values that lie between 0 and 10?

(B)

4-40. There were two exams in an introductory statistics course. The scores on Exam 1 followed a normal curve with mean 60 and standard deviation 10 and on Exam 2 the scores followed a normal curve with mean 80 and standard deviation 5. Jon scored 80 on each exam. Find Jon's percentile scores on each. Comment.

Problem 4.

(A)

4-42. The weight of anodized reciprocating pistons produced by Brown Company follows a normal distribution with mean 10 lbs. and standard deviation 0.2 lb.

a. The heaviest 2.5% of the pistons produced are rejected as overweight. What weight, in pounds, determines the overweight classification? Give your arguments.

b. Suppose Brown Company can sell only those pistons weighing between 9.8 and 10.4 lbs.. What proportion of the pistons is lost?

c. If a sample of pistons is available, how might you answer a. and b. without the normal population model?

(B)

4-43. Let Y represent the number of years that a randomly chosen Ph.D. on the faculty spent obtaining his or her doctorate. Assume that Y is normally distributed with a mean of 5.8 years, and a variance of 6.25 years squared. What proportion of the faculty members spent more than 9 years obtaining their doctorate?

(C)

4-44. A paint company knows from experience that the area in square feet, that one gallon of its premium paint will cover follows a $N(250, 25)$ distribution model. The company wishes to advertize that if a gallon of this paint fails to cover t square feet, it will refund the purchase price plus 10%. What is the largest value of t that will ensure that on average no more than 0.5% of all purchases will be given the refund?

Problem 5.

(20pts) A fair coin is independently tossed three times until a head appears, whichever comes first. Let X denotes the number of heads.

(A) Write the sample space with associated probabilities for outcomes, and then find the probability distribution of X.

(B) What is the expected number of heads.

Problem 6.

(20pts) A fair coin is independently tossed three times until a head appears, whichever comes first. Let X denotes the number of tails.

> **(A)** Write the sample space with associated probabilities for outcomes, and then find the probability distribution of X.
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> **(B)** What is the expected number of tails.